

MODEL Question based on revised syllabus

Tribhuvan University

Four Years Bachelor Level /Science and Technology

Full Marks: 100

Pass Marks: 35

Probability and Inference – I (STA201)

Time =3 hours

B.Sc. II Year

Group A

Attempt any **FOUR** questions. [4x10 =40]

1. Define Negative binomial distribution. Derive its mean and variance.
2. Explain Gamma distribution. Show that a gamma distribution with parameter α tends to Normal distribution as $\alpha \rightarrow \infty$ (i.e. for large value of parameter α)
3. Define the probability density function and distribution function for bivariate random variable. Write down the properties of bivariate distribution function.
4. Derive student's t- distribution. If F follows F distribution with(m, n) degrees of freedom then show that F(m,n) distribution converted to t- distribution when $m=1$.
5. Define the method of maximum likelihood estimation. What are the properties of maximum likelihood estimators?
6. Differentiate between parametric and non parametric test. Explain the process of carrying out one sample run test with suitable example.

Group B

Attempt any **Eight** questions. [8x5=40]

7. Obtain the moment generating function of negative exponential distribution and, find its mean and variance.

8. If X is distributed as a beta distribution of first kind with parameter $m=3$ and $n=4$, then find mean, mode and variance of the beta distribution.

9. If X_1 and X_2 are two independent rectangular variates on $[0, 1]$, find the distribution of $X_1 X_2$.

10. The joint distribution of X and Y is given by

$$f(x, y) = 4xy e^{-(x^2+y^2)}, \quad x \geq 0, y \geq 0 \\ = 0, \quad \text{otherwise.}$$

Examine whether random variables X and Y are independent?

11. Define likelihood function, and prove its properties.

12. Examine whether Minimum Variance Bound (MVB) estimator of μ exists or not, when the 'n' sample observations are taken from $N(\mu, \sigma^2)$ population, if it does find MVB.

13. Define interval estimation. If T is unbiased estimator of θ , then show that T^2 is a biased estimator of θ^2 .

14. State and prove the Neyman-Pearson's lemma. Also write its applications.

15. Obtain the values of type I and type II errors, if $x \geq 1$ is the critical region for testing $H_0: \theta = 2$ against the alternative hypothesis $H_1: \theta = 1$, on the basis of a single observation from the population

$$f(x, \theta) = \theta e^{-\theta x} \quad 0 \leq x < \infty,$$

16. Two groups of rats, one group consisting of trained ones, another groups not trained one have the following number of trials to achieve certain criterion:

Trained rats	78	64	75	45	82
Untrained rats	110	70	53	51	

Use Mann- Whitney U test if there is a difference between the two average number of trials of trained and untrained rats.

Group C

18. Attempt **ALL** questions. [10x2=20]

- a) Find the standard error of sample proportion.
- b) What are one tailed test and two tailed tests in testing of hypothesis?
- c) Give an example for the outcome of a random experiment that is two dimensional random variable.
- d) A plant produces steel sheets whose weights are normally distributed with a standard deviation of 2.4 kg. A sample of 10 had a mean weight of 31.4 kg. Find 95% confidence limits for the population mean.
- e) What are the four main features of F- distribution curve?
- f) Write down mean and variance of hyper geometric distribution.
- g) Write down the recurrence relation of Chi –Square distribution on with ‘n’degrees of freedom.
- h) Give the statement of Cramer - Rao’s Inequality.
- i) What are the characteristics of good estimator?
- j) Give moment generating function of Cauchy distribution.